SYNOPSIS

Title of Research:

On ordered semirings and its decompositions

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In 2002, Kehayopulu and Tsingelis [6] gave the decomposition theorem of the left regular ordered semigroups into left regular and left simple semigroups, and in 2013 [5], they studied the structure of ordered semigroups which are semilattices of left simple semigroups via Green's $\mathcal{L}$-relation, where it is found that the relation $\mathcal{L}$ is the least semilattice congruence on S. In 2012, Bhuniya and Mondal [2] studied the structure of semirings with semilattice additive reduct which are distributive lattices of left

k-simple and k-simple semirings, where the notion of k-ideal played a vital role in such decompositions. If $\mathcal{C}$ is a class of semirings, refering to semirings in $\mathcal{C}$ we say that a semiring S is called a distributive lattice of $\mathcal{C}$-semirings if there is a congruence ρ on S such that S/ρ is a distributive lattice and each ρ-class is a semiring in $\mathcal{C}$. Clifford and Preston studied the structure of semigroups [3] and gave decompositions of such semigroups into Archimedean components. In 2006, Keyahopulu and Tsingelis [4] decomposed a commutative ordered semigroup $(S, \cdot, \leq)$ into Archimedean components by a relation η on S given by: $a\eta b \Leftrightarrow a|b^n$ and $b|a^m$ for some $m, n \in \mathbb{N}$. Here $a|b$ if there is $x \in S^1$ such that $b \leq ax$. Motivating from the notion of semilattice decomposition of semigroups, Bhuniya and Mondal [1] introduced the notion of k-Archimedean semirings, and studied the distributive lattice decomposition of semirings with semilattice additive reduct into simple components such as k-Archimedean semirings. Keeping in mind all the above introductions of new concepts in semigroups, ordered semigroups and success of introducing analogous concept in semirings with semilattic additive reduct, we are interested to generalize and expect to give some analogous concepts of simplicity, regularity and decomposition theory in ordered semirings.

If $(S, +, \cdot)$ be a semiring and $(S, \leq)$ a partially ordered set, then $(S, +, \cdot, \leq)$ called an ordered semiring[7] if for $a, b, c \in S, a \leq b \Rightarrow a + c \leq b + c, c + a \leq c + b, ac \leq bc, ca \leq cb$. The algebraic structure $(\mathbb{N}, +, \cdot, \leq)$ is an example of ordered semiring, where $\leq$ is the usual 'less than or equal to' relation. As in the case of distributive lattice decompositions in semirings, the Green's relations play an important role, we expect the same in ordered semirings too. We are also curious about the description of the least distributive lattice congruence on an ordered semiring.

References

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